NATIONAL MUSEUM OF THE UNITED STATES AIR FORCE

MATHEMATICS OF FLIGHT: HEADWINDS AND TAILWINDS

Students will have a basic understanding of math applications used in flight. This includes the effects of headwinds and tailwinds on aircraft speed. Students will solve a series of problems. (One in a series)

LESSON PLAN

Grade Level: High School-Algebra

Common Core State Standards for Mathematics:
Algebra: Seeing Structure in Expressions; Creating Equations and Reasoning with Equations and Inequalities

Technology Content Standards (from STL):
Technology and Society

Lesson Objectives

The students will:

- Be introduced to formulas used in flight related to navigation and aircraft performance.
- Learn to calculate the effects of headwinds and tailwinds on speed.

Goal

In this lesson, students will gain an understanding of common calculations performed by flight personnel.

Headwinds and Tailwinds

Headwinds are winds blowing against the direction of travel; tailwinds are blowing in the direction in which an aircraft is traveling. A tailwind adds to an aircraft's effective speed.

Example:

With a tailwind, an aircraft travels 180 miles in 45 minutes. With a headwind, it travels 180 miles in one hour. Find the speed of the aircraft in still air (p).

Materials Required:

- Paper
- Pencil or pen
- Calculator (optional)

Solution:

When you have constant speed, the formula for uniform motion is: **distance = rate X time**
Since knots are nautical miles per hour, the first step would be to change the minutes to hours.

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	p + w	X	¾ hour	=	¾(p+w)
Head-wind	p - w	X	1 hour	=	p - w

$$45 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{45 \div 15}{60 \div 15} = 3/4 \text{ hour}$$

In the chart above, **p** represents the speed of the aircraft in still air; **w** represents the speed of the wind.

(Note: Because the distance in both situations (headwind and tailwind) is 180 miles, the two equations for distance can be set equal.)

$$
\begin{aligned}
3/4\,(p + w) &= p - w \\
4[3/4\,(p + w) &= 4\,(p - w) \\
3\,(p + w) &= 4\,(p - w) \\
3p + 3w &= 4p - 4w \\
3w &= 4p - 4w - 3p \\
3w + 4w &= p \\
7w &= p
\end{aligned}
$$

The aircraft traveled 180 nautical miles with the tailwind and 180 nautical miles with the head wind. Use either distance from the chart on the previous page equal to 180, substituting 7w for **p** to find **w**, the speed of the wind.

$$
\begin{aligned}
p - w &= 180 \\
7w - w &= 180 \\
6w &= 180 \\
\frac{6w}{6} &= \frac{180}{6} \\
w &= 30
\end{aligned}
$$

$$
\begin{aligned}
3/4(p + w) &= 180 \\
3/4(7w + w) &= 180 \\
3/4\,(8w) &= 180 \\
6w &= 180 \\
\frac{6w}{6} &= \frac{180}{6} \\
w &= 30
\end{aligned}
$$

The wind speed is 30 knots.

To find the speed of the aircraft, use either distance from the chart and substitute 30 for **w.**

$$
\begin{aligned}
p - w &= 180 \\
p - 30 &= 180 \\
p &= 180 + 30 \\
p &= 210
\end{aligned}
$$

$$
\begin{aligned}
3/4(p + w) &= 180 \\
4[3/4(p + 30)] &= (180)4 \\
3(p + 30) &= 720 \\
\frac{3(p + 30)}{3} &= \frac{720}{3} \\
p + 30 &= 240 \\
p &= 240 - 30 \\
p &= 210
\end{aligned}
$$

The speed of the aircraft is 210 knots.

Exercise 1:
With a tailwind, an aircraft travels 127 nautical miles in 30 minutes. With a headwind, it travels 107 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

The first step is to convert minutes to hours.

$$30 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{30 \div 30}{60 \div 30} = 1/2 \text{ hour}$$

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	p + w	X	1/2 hour	=	127
Headwind	p - w	X	1/2 hour	=	107

(**p** represents the speed of the aircraft in still air; **w** represents the speed of the wind.)
Because the times are the same in this exercise, represent the times in terms of distance and set the algebraic representations equal to each other in the equation. The formula for uniform motion is:
distance = rate X time

$$\frac{127}{p + w} = \frac{107}{p - w}$$

$$\begin{aligned}
127 (p - w) &= 107 (p + w) \\
127p - 127w &= 107p + 107w \\
127p - 127w - 107p &= 107w \\
20p - 127w &= 107w \\
20p &= 107w + 127w \\
20p &= 234w \\
\frac{20p}{20} &= \frac{234w}{20} \\
p &= 11.7w
\end{aligned}$$

Substitute **p** in the chart to find the wind speed.

$$\begin{aligned}
1/2(p + w) &= 127 \\
1/2(11.7w + w) &= 127 \\
1/2 (12.7w) &= 127 \\
1/2 (12.7w) \times 2 &= 127 \times 2 \\
12.7w &= 254 \\
\frac{12.7w}{12.7} &= \frac{254}{12.7} \\
w &= 20
\end{aligned}$$

The wind speed is 20 knots.

To find the speed of the aircraft in still air, substitute the wind speed in the chart.

$$\begin{aligned}
1/2(p + w) &= 127 \\
1/2(p + 20) &= 127 \\
1/2(p + 20) \times 2 &= 127 \times 2 \\
p + 20 &= 254
\end{aligned}$$

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	p + w	X	1/2 hour	=	127
Headwind	p - w	X	1/2 hour	=	107

To verify the aircraft speed and the wind speed, substitute them in the chart, using the headwind.

$$1/2(p - w) = 107$$
$$1/2(234 - 20) = 107$$
$$1/2\ (214) = 107$$
$$107 = 107$$

Exercise 2:
With a headwind, an aircraft travels 300 nautical miles in 45 minutes. With a tailwind, the aircraft travels 230 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

The first step is to convert minutes to hours.

$$45 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{45 \div 30}{60 \div 30} = 3/4 \text{ hour}$$

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	p + w	X	1/2 hour	=	230
Headwind	p - w	X	3/4 hour	=	300

By solving for **p** or **w** using the headwind data, we can then substitute the result in the tailwind rate and find the aircraft and wind speeds.

$$3/4(p - w) = 300$$
$$3/4(p - w) \times 4 = 300 \times 4$$
$$3\ (p - w) = 1200$$
$$\frac{3\ (p - w)}{3} = \frac{1200}{3}$$
$$(p - w) = 400$$
$$p = w + 400$$

Using the tailwind data:

$$1/2(p + w) = 230$$
$$1/2(w + 400 + w) = 230$$
$$1/2(2w + 400) = 230$$
$$1/2(2w + 400) \times 2 = 230 \times 2$$
$$(2w + 400) = 460$$
$$(2w + 400 - 400) = 460 - 400$$
$$2w = 60$$
$$\frac{2w}{2} = \frac{60}{2}$$
$$w = 30$$ The wind speed is 30 knots.

Solve the above: $p = w + 400$
$p = 30 + 400$
$p = 430$ The aircraft is traveling 430 knots in still air.

Exercise 3:
With a tailwind, an aircraft traveling at 400 miles per hour can fly 220 nautical miles in 30 minutes. With a head-wind, the aircraft can only fly 180 nautical miles in 30 minutes, What is the speed of the wind?

The first step is to convert minutes to hours.

$$30 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{30 \div 30}{60 \div 30} = 1/2 \text{ hour}$$

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	400 + w	X	1/2 hour	=	220
Headwind	400 - w	X	1/2 hour	=	180

Solve for **w** using the headwind data:

$$1/2(p - w) = 180$$
$$1/2(400 - w) = 180$$
$$1/2(400 - w) \times 2 = 180 \times 2$$
$$(400 - w) = 360$$
$$(400 - w) + w = 360 + w$$
$$400 = 360 + w$$
$$400 - 360 = w$$
$$40 = w$$

Using the tailwind data:

$$1/2(p + w) = 220$$
$$1/2(400 + w) = 220$$
$$1/2(400 + w) \times 2 = 220 \times 2$$
$$(400 + w) = 440$$
$$(400 + w - 400) = 440 - 400$$
$$w = 40$$

The wind speed is 40 knots.

See student worksheet and presentation.

Resources:

National Museum of the United States Air Force

Belcher, Diana. *Education in Flight: A Teacher's Guide to the Mathematics of Flight.* Department of the Air Force, 2007.

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MATHEMATICS OF FLIGHT: HEADWINDS AND TAILWINDS

STUDENT WORKSHEET *NAME:*_____

When you have constant speed, the formula for uniform motion is: distance = rate X time

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	p + w	X		=	p + w
Headwind	p - w	X		=	p - w

Exercise 1:
With a tailwind, an aircraft travels 127 nautical miles in 30 minutes. With a headwind, it travels 107 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

Exercise 2:
With a headwind, an aircraft travels 300 nautical miles in 45 minutes. With a tailwind, the aircraft travels 230 nautical miles in 30 minutes. Find the speed of the aircraft in still air.

MATHEMATICS OF FLIGHT: HEADWINDS AND TAILWINDS—CONTINUED

STUDENT WORKSHEET

Chart	Rate	X	Time in Hours	=	Distance
Tailwind	400 + w	X		=	220
Headwind	400 - w	X		=	180

Exercise 3:
With a tailwind, an aircraft traveling at 400 miles per hour can fly 220 nautical miles in 30 minutes. With a headwind, it can only fly 180 nautical miles in 30 miles. What is the speed of the wind?